"…drawn from the bosom of nature herself": The Trumpet, the Trumpet Marine, and the Discovery of the Harmonic Series

Stewart Carter

In the early stages of the Scientific Revolution, the trumpet and the trumpet marine ("bowed monochord") were often invoked in attempts to explain the phenomenon we now call the harmonic series. In this article I trace the modern discovery of this acoustical phenomenon, beginning with Marin Mersenne's study of harmonics and culminating in the work of Francis Robartes and Joseph Sauveur, showing how an understanding of the natural notes of these two instruments aided in this discovery.

Marin Mersenne

The Minorite friar Marin Mersenne (1588–1648) was the first scholar to examine harmonics systematically.[1] He initially mentioned hearing harmonics in his *Quaestiones celeberrimae in Genesim* (1623):

> I have ... observed that in the sound of one bell three musical parts are produced, the bass, which was the primary and proper sound, the fifth, and the octave; I think that I have sometimes distinguished even the fourth and the major third. And I believe that this can be observed in organ pipes as well, and in other instruments, … even in the [human] voice.[2]

The partial frequencies of bells do not follow the simple pattern of intervals typically associated with the harmonic series. It is unlikely, however, that Mersenne heard a *simple* fourth or fifth above the bell's fundamental or "strike" tone, though he may have heard their compounds.[3]

Mersenne was an avid correspondent, communicating frequently with scholars throughout Europe in his search for answers to scientific, theological, and musical questions. In 1634 or 1635 he circulated to some of his friends a list of questions relating to the theory of sound, three of which are particularly relevant to this discussion:

> 1. Why does one string of an instrument, without being pressed by the finger in order to shorten it, make several sounds, and wind instruments without opening or closing any holes [do the same]?
> 2. Why do string and wind instruments, without pressing the string on the neck or opening or closing any holes, skip from the lowest tone first to the octave, then to the twelfth, then to the fifteenth?

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> 3. Why in the bass or low range of the trumpet can one not intone *ut, re, mi, fa, sol, la*?[4]

In Mersenne's *Harmonie universelle* (1636), Proposition IX in the Fourth Book of String Instruments is entitled "To determine why plucking an open string creates many sounds at the same time."[5] Here he states,

> Aristotle … posed the question why a low sound contains the higher [octave]….[6] But … he did not know that an open string when struck makes at least five different tones at the same time, of which the first is the natural sound of the string, which serves as the foundation to the others....
>
> These sounds follow the ratios … 1, 2, 3, 4, 5, because one hears four sounds different from the natural tone [i.e., the fundamental], of which the first is the higher octave; the [next], the twelfth; [then] the … fifteenth, and … the major seventeenth…. [T]hese tones follow the same progression as the leaps of the trumpet.
>
> Apart from these four … tones, I hear a fifth one still higher, which I hear particularly toward the end of the natural sound, and at other times a little after the beginning; it produces the major twentieth [i.e., an octave and a sixth] with the natural tone.[7]

Probably what Mersenne heard as a major twentieth was actually the out-of-tune seventh harmonic, which is closer to a minor twenty-first, or compound minor seventh.

Mersenne and the trumpet marine

The trumpet marine (Fr., *trompette marine*; It., *tromba marina*) is an instrument typically with a single melody string and a bridge with one vibrating foot. The player does not depress the string firmly against the body of the instrument, but touches it lightly, bowing between her/his finger and the nut. Every natural note is therefore a harmonic, so the instrument is entirely dependent on eliciting notes of the harmonic series.[8] Mersenne remarks that the succession of notes on the trumpet marine is like that of the military trumpet.

> These skips … that imitate the tones of the military trumpet are nothing more than a convincing explanation of what the string makes when it is played open; that is to say, the octave, twelfth, fifteenth, seventeenth, nineteenth, etc., … which it produces at the same time, altogether.[9]

Mersenne provides two drawings of the marine trumpet, one with a second string acting as a drone (Figure 1).[10] The letters on the right side of the one-string instrument and those on the right side of the two-string instrument apparently are not intended as indications for finger placement, since many of them do not represent aliquot (i.e., whole-number fraction) divisions of the string. For the one-string instrument, the numbers may represent finger placement for obtaining a scale on a "normal" string instrument without a vibrating bridge.[11]

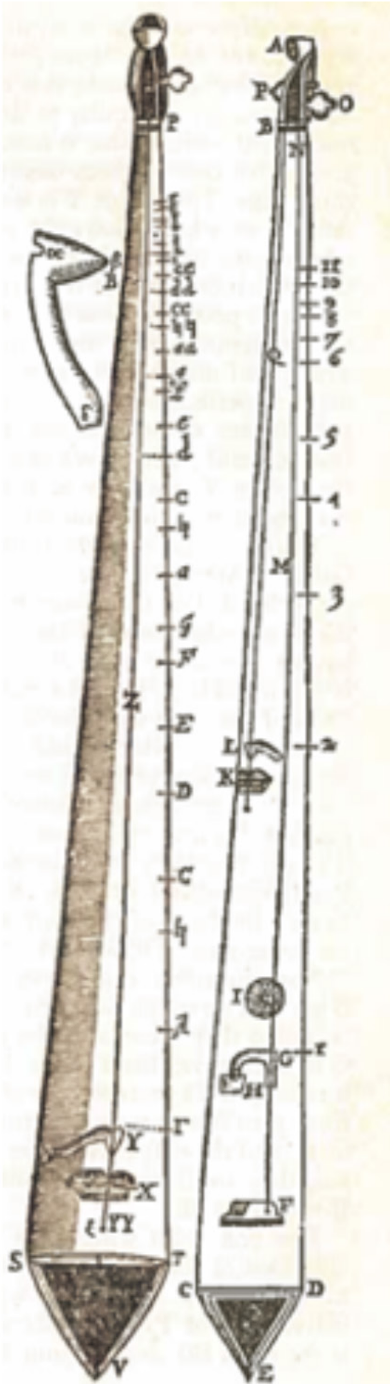


Figure 1: Two trumpets marine. Marin Mersenne, *Harmonie universelle* (1636), 3:218.

Mersenne and the trumpet

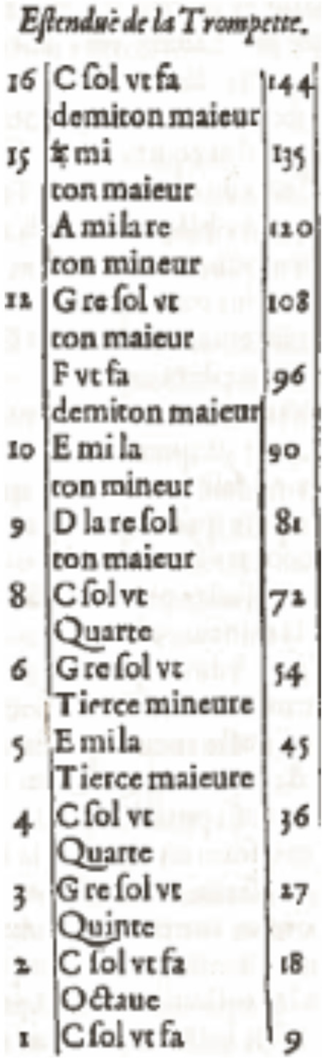
Mersenne was thus aware of the similarities between the notes of the trumpet and the trumpet marine, but he could not explain the "skips" he observed in both instruments. Turning once again to Mersenne's *Quaestiones celeberrimae in genesim* (1623), we read,

> I shall mention here … an observation made concerning the *tuba* [i.e., trumpet] by J[ehan] Titelouze[12] …, that if anyone plays … the lowest sound, if he should wish thereafter to play it higher than before, [it ascends] by the octave; … if he should wish to surpass the octave, it [ascends] by the fifth; if he surpasses the octave [and] the fifth, it ascends by a fourth, then by the major third, the minor third, the major second, the minor second; from this [Titelouze] … concludes that the natural succession … of those intervals is *drawn from the bosom of nature herself* [italics mine].[13]

Titelouze thus came close to describing the "natural" harmonic series, though he erred in placing both the major second and minor second immediately after the sixth harmonic.[14]

Since these notes are "drawn from the bosom of nature," Mersenne could not bring himself to admit dissonant tones, so his chart of trumpet notes (Figure 2) omits the seventh harmonic. Since the seventh harmonic of a series built on *C* is a note that falls "between the cracks"—i.e., it is a flat *b-flat*^{1}—Mersenne rejected it because it creates a dissonance with the fundamental. Yet Mersenne accepted the ninth harmonic, d^2, because it makes a perfect fifth with the sixth harmonic, g^1. In his view, then, the ninth harmonic is satisfactory because it forms a consonance with a note below it, though not with the fundamental, whereas the seventh harmonic is unacceptable because it does not form a consonance with any lower note. With equally specious reasoning, he explains that the eleventh harmonic, f^2 (which is not assigned a number in the left-hand column of his chart; see Figure 2), as a consonance—a perfect fourth—with the eighth harmonic. This fourth, however, would be quite out-of-tune. Mersenne rationalizes his explanations by pointing out that the "true" consonances appear in the lower-numbered harmonics, while the "compound" consonances (such as that of the ninth harmonic) appear only after the eighth harmonic has been reached.

Mersenne's chart of trumpet pitches (Figure 2) illustrates how closely he relied on just intonation. His choice of the value of 9 to represent the lowest trumpet note, *C*, is arbitrary, but it allows for a major whole tone (9:8 ratio; value of 81 in Figure 2) between c^2 and d^2 and a minor whole tone (10:9 ratio; value of 90 in Figure 2) between d^2 and e^2. But while the ratio of f^2 to e^2 is given correctly—96:90, or 16:15—for a minor second in just intonation, his system of multiples by a factor of 9 breaks down here. It also breaks down between harmonics 12 and 15, where Mersenne has placed only one note, a^2, though *two* different harmonics, numbers 13 and 14, should appear here. However, neither would be in tune according to just intonation.



Estenduë de la Trompette.

16	C sol vt fa	144
	demiton maieur	
15	♮ mi	135
	ton maieur	
	A mi la re	120
	ton mineur	
12	G re sol vt	108
	ton maieur	
	F vt fa	96
	demiton maieur	
10	E mi la	90
	ton mineur	
9	D la re sol	81
	ton maieur	
8	C sol vt	72
	Quarte	
6	G re sol vt	54
	Tierce mineure	
5	E mi la	45
	Tierce maieure	
4	C sol vt	36
	Quarte	
3	G re sol vt	27
	Quinte	
2	C sol vt fa	18
	Octaue	
1	C sol vt fa	9

Figure 2: Range of the trumpet. Marin Mersenne, *Harmonie universelle* (1636), 3:249.

It should be noted that, as Myers, Campbell, and Gilbert state,

> [T]he pitches of the playable notes [of the trumpet] are strongly influenced by the resonance frequencies of the instrument's air column…. [I]n realistic brass instruments these resonance frequencies are never exact members of a harmonic series. For that reason we … avoid using the term 'harmonic' to describe one of the easily playable notes on a brass instrument, employing instead the more neutral term 'natural note'.[15]

Mersenne included two illustrations of a twice-folded natural trumpet in *Harmonie universelle*, shown in Figures 3 and 4. Figure 3 is a simple drawing of a trumpet with two mouthpieces, one inserted in the instrument and the other beside it. Circumscribed in the cord above the instrument in Figure 4 is a chart of ranges, while another chart appears in the banner below. The author sometimes ignores the octave traditionally implied by these clefs and it is unclear why in the chart of ranges below the instrument appears to reflect a harmonic series on *F* (at least, after the first four notes), though the intervals beside each note appear to represent their intervals above *C*.

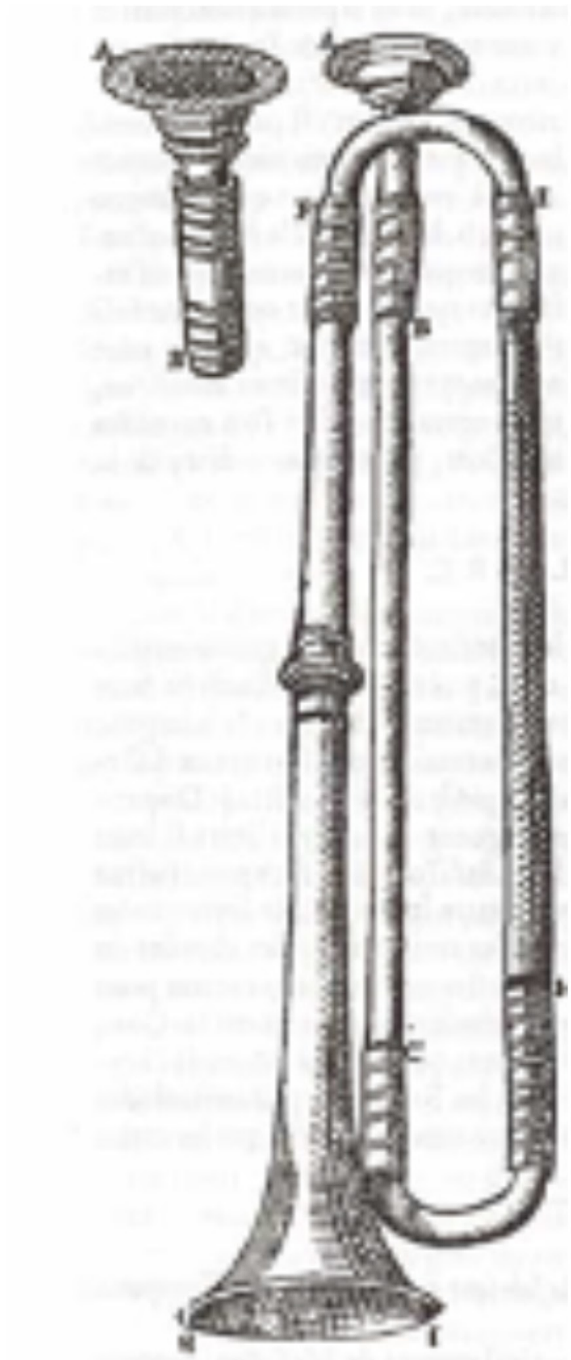


Figure 3: Twice-folded trumpet, with mouthpieces. Mersenne, *Harmonie universelle* 3:248.

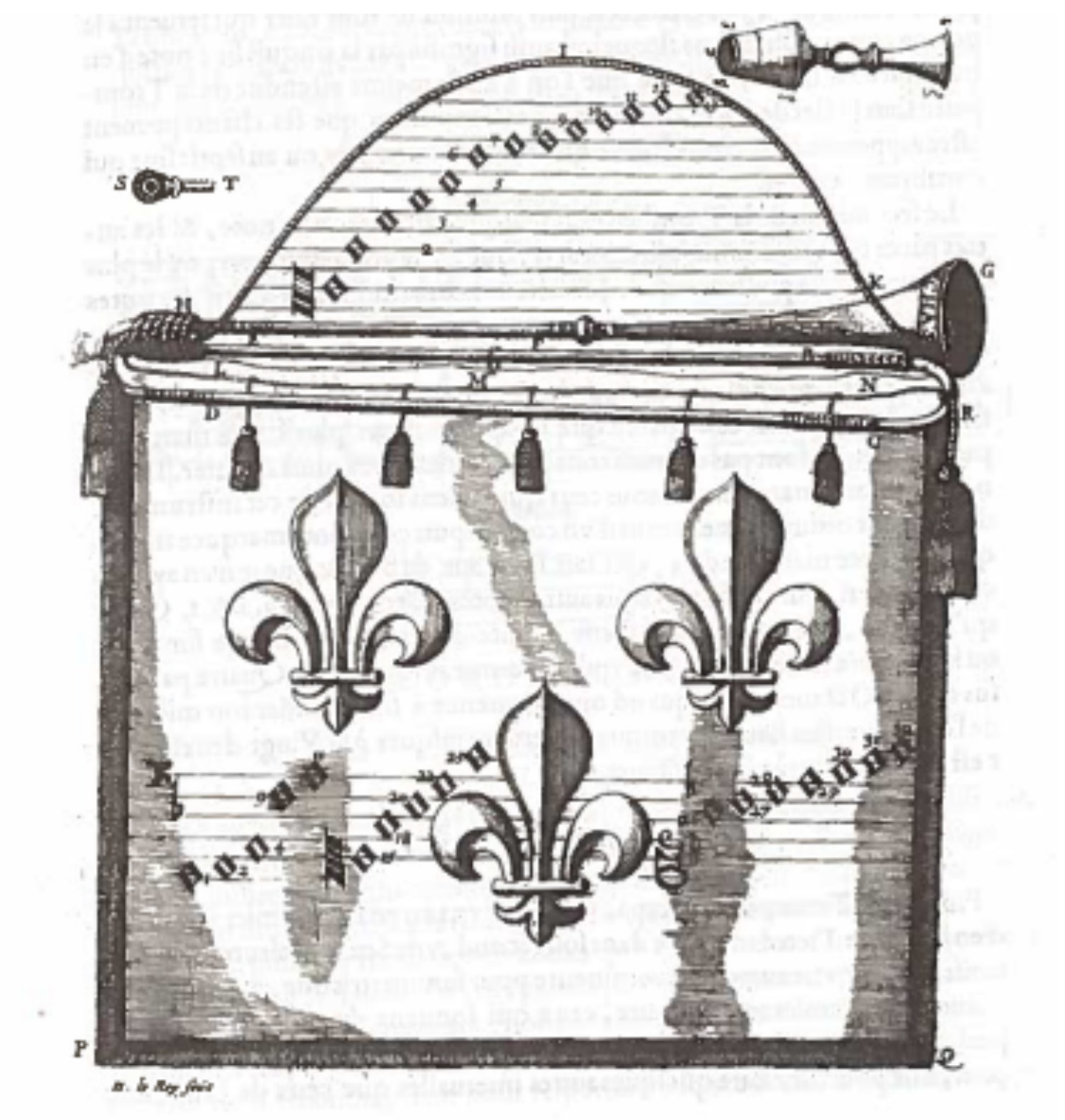


Figure 4: Folded trumpet, banner, and range charts. Mersenne, *Harmonie universelle*, 3:267.

New acoustical discoveries in England

Four Oxford scholars: Noble, Pigot, Marsh, and Wallis

In 1673 or 1674 some scholars in Oxford made an important acoustical discovery: the existence of nodes in vibrating strings. In 1677 Narcissus Marsh, principal of St. Alban Hall, wrote an account of this discovery for Robert Plot's *Natural History of Oxford-shire,* ascribing it to William Noble and Thomas Pigot.[16] In that same year John Wallis, Savilian Professor of Geometry at Oxford, reported the discovery in a letter to Henry Oldenburg, First Secretary of the Royal Society. The letter, which was published in *Philosophical Transactions of the Royal Society* for 1677, mentions Noble and Pigot, but ignores Marsh's report, even though Wallis clearly knew of it and even made a heavy-handed attempt to "scoop" Marsh.[17]

The available evidence suggests that this discovery, which provides the first lucid explanation of nodes in vibrating strings, probably should be credited to Noble, though Pigot may have come to the same conclusions independently. According to Anthony à Wood, Noble kept the discovery to himself, imparting it only to a few friends, while Pigot, "being a more forward and mercurial man got the glory of it among most scholars."[18] Marsh's illustrations, as shown in Plot's book (see Figure 5), are confusing because they show strings of the same length that are not of the same pitch. Figures 6 and 7 revise two of these illustrations to represent the lengths of the strings proportionally.

Figure 5: Robert Plot, *History of Oxford-shire* (1677), Tab. XV, nos. 1–13.

Marsh's Proposition 2, as reported by Plot, is particularly relevant to the topic at hand:

> *If the lesser of two octaves*, BC, *be touched … each half of the greater,* C2, 2D, *will answer it, the middle,* 2, *standing still….* About *CD* wrap loosely three narrow *strips* of *paper,* one in the middle, 2, the other betwixt *C*2 and 2*D* [i.e., at *p* and *q*] … then with the *finger* or *bow* strike *BC*, or any part of it, and you will see the papers in *p q* dance up and down and about the string, twixt *C*2 and 2*D*, but that in 2 stand still. Whence it is evident that *CD* moves in two *halves*, by two *distinct* motions [see Figure 6] [italics original].[19]

Marsh's Proposition 4:

> *If the lesser of two fifths*, CD, *be toucht on either of its halves* C2, 2D, *each third part of the greater* DX, XZ, ZE, *will answer it, but* [if you strike the string] i*n*

the middle, 2, *they will not stir*. Which will plainly appear, By laying *papers* as before, on *t*, *x*, *3*, *z*, *v*, if then you strike *C2* or *2D*, you'l see the papers on *t*, *3*, v, frisk and daunce, while those on x and z stand still, but if you strike it on 2 none will move [see Figure 7] [italics original].[20]

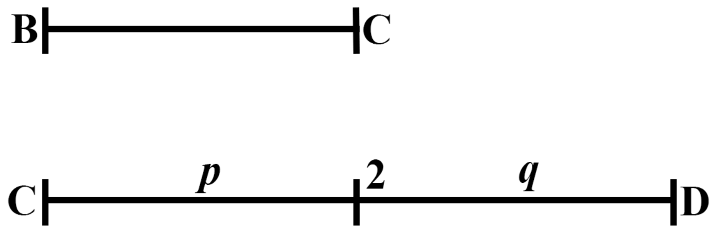


Figure 6: Two strings, tuned an octave apart. After a diagram, probably by Narcissus Marsh, in Robert Plot, *History of Oxford-shire*, unnumbered page after 328.

The crucial point is that the strips of paper attached at the proper points of the string that is not plucked or bowed will flutter conspicuously if they are at displacement antinodes but will not move if they are positioned at displacement nodes.

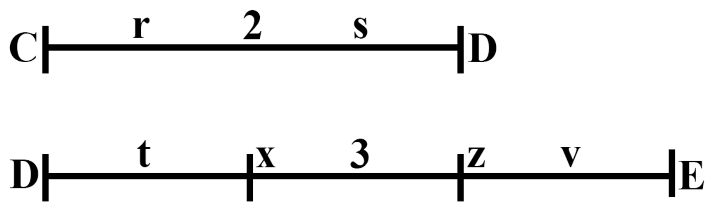


Figure 7: Two strings, tuned a perfect fifth apart. After a diagram, probably by Narcissus Marsh, in Plot, *The Natural History of Oxford-shire*, unnumbered page after 328.

Francis North

Significant as they are for the history of the science of acoustics, the experiments of these Oxford scholars do not relate directly to the trumpet or the trumpet marine, though a contemporary of theirs, Francis North, writing in the same year as Plot's book and Wallis's letter to Oldenburg, mentioned both instruments. The trumpet marine apparently had only recently become somewhat well known in England; Samuel Pepys's description of a private performance on the instrument in 1667 by "Monsieur Prim" is possibly the earliest reference to the instrument in an English source.[21] North did not conduct experiments like those of Noble and Pigot, but in his *Philosophical Essay on Musick* (1677), he wrote about strings that "break into chords above," a rather oblique reference to harmonics.

> It is common experience, that a great string struck near the Bridge with a Bow where the Rosin takes but small hold, will whistle and break into chords above; which if it were struck by the thumb that removes it out of its place, would give the true *Tone.*
>
> The *Trumpet marine* that sounds wholly upon such breaks, is a large and long monochord play'd on by a Bow near the end, which causes the string to break into shrill Notes. The removing the thumb that stops upon the string gives measure to these breaks, and consequently directs the *Tone* to be produced in imitation of a *Trumpet,* which otherwise would be like a *Whistle* or *Pipe.*
>
> The touch of the Thumb less hinders the *Sound* of the string when it is upon the point where the vibrations cross, than when it is in any other part: for we see when any great string has an entire vibration, such a touch would immediately extinguish the *Sound.* This makes the *Trumpet Marine,* with the Thumb placed upon it, take to such a Note, as that the division of vibrations shall lye just under the Thumb.
>
> Speaking of the counterfeit *Trumpet* [i.e., the trumpet marine], I must observe that the true one [i.e., the ordinary trumpet] seems to give all its Notes by way of breaking, which causing the metal to jarr gives so loud a sound …. And for the rest of the Notes which a [brass] *Trumpet* will easily produce, they are the third ♯, fourth, fifth and sixth sharp [i.e., the flat seventh harmonic], which arise upon the most easie divisions of the monochord, and therefore most readily produced by breaking, when the strength of the blast and the action of the lips direct it[22] [italics original].

Francis Robartes (Roberts)

The next significant step in the discovery of the harmonic series was made by Francis Robartes, an English politician, mathematician, composer, and member of Parliament.[23] He joined the Royal Society in 1673 and thus may have been present when John Wallis's letter to Henry Oldenburg was read before the Society on 25 March 1677; in any case, he certainly was familiar with the Oxford experiments with vibrating strings. His "Discourse Concerning the Musical Notes of the Trumpet, and Trumpet Marine, and the Defects of the Same" appeared in the *Philosophical Transactions* for 1692.[24] In this essay he poses two basic questions concerning the trumpet:

> 1. Whence it comes to pass that the Trumpet will perform no other Notes … but only those … which are usually called by Musicians Trumpet-Notes.
> 2. What is the reason that the 7th, 11th, 13th, and 14th Notes are out of Tune, and the others exactly in Tune.[25]

Robartes's staff notation in Figure 8 offers by far the most accurate representation of the natural tones available on a trumpet in C to appear up to 1692. As Robartes notes, "take notice that the Prickt notes [i.e., those shown with dotted lines] are imperfect, not being exactly in tune, but a little flatter or sharper than the places where they stand, according as *f* or *s* is set over them."[26]

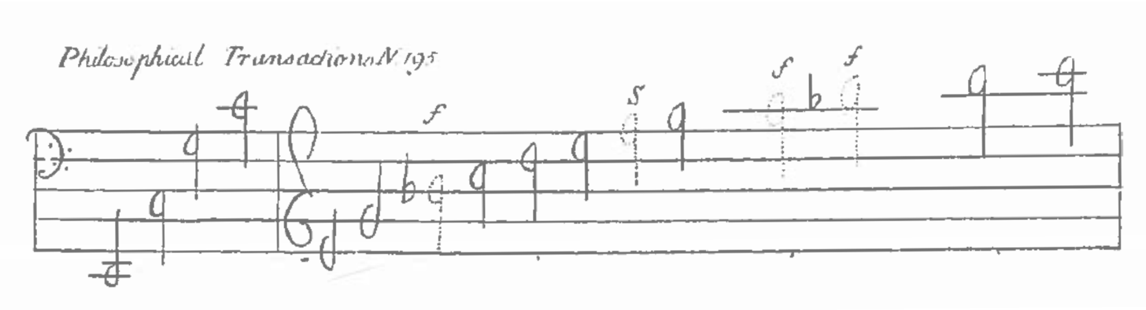


Figure 8: Trumpet notes, from Francis Robartes, "A Discourse concerning the Musical Notes of the Trumpet, and Trumpet Marine," *Philosophical Transactions of the Royal Society* 16 (1692), unnumbered page after 563. In this chart of harmonics, *f* = flat; *s* = sharp.

Perhaps following Mersenne, Robartes then invokes the trumpet marine by way of comparison, noting that it produces the same intervals as the trumpet. He proceeds to explain that a string tuned to some aliquot part of a second string will vibrate sympathetically when the second string is struck. This is because, he says, a string vibrates, not only as a whole, but also in parts. But if a second string is tuned to *f* it will not vibrate in sympathy with the string tuned to *c* because the vibrations of the first string (tuned to *f*) do not match an aliquot part of the second string.
Robartes obviously knew of the experiments described in Wallis's report, for the illustrations in Figure 9 are based on earlier diagrams, as is his description of nodes in vibrating strings:

> If a Unison is struck, it makes one intire vibration in the whole String, as in Fig. *A,* and the motion is most sensible in the middle at *m,* for there the vibrations take the greatest scope.
>
> If an 8th. is struck, it makes two vibrations, as in Fig. *B,* and then the point *m* is in a manner quiescent, and the most sensible motion at *n, n.*
>
> If a 12th. [the interval between strings C and D] be struck, then it makes three vibrations, as in [the illustration], and the greatest motion [will be] at *q, m, q,* and hardly to be perceived at *p, p.* All which may be plainly experimented by putting a little piece of Paper upon the several parts of the String to make the motion more conspicuous.[27] [see Figure 9].

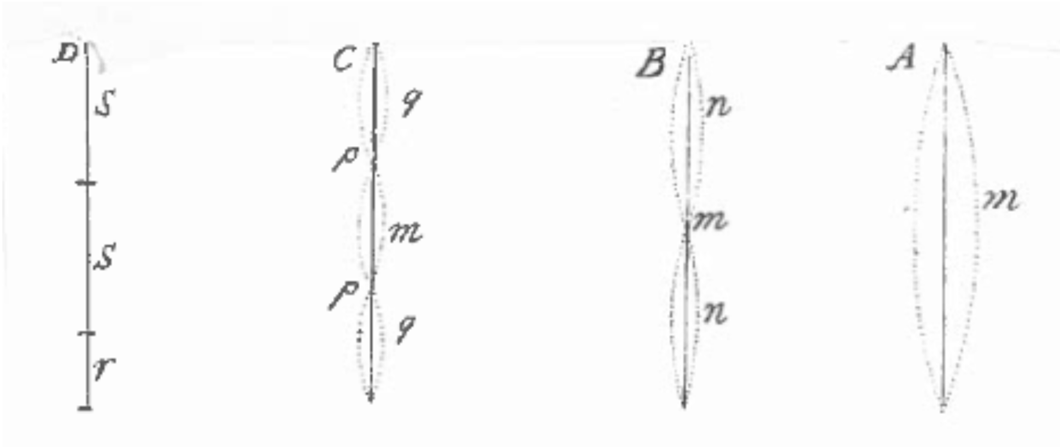


Figure 9: Sympathetic vibrations and nodes in vibrating strings. Robartes, "Discourse," unnumbered page after 563.

Robartes further observes that the trumpet marine

> will yield no Musical sound but when the stop makes the upper part of the String an *aliquot* of the remainder, and consequently of the whole; otherwise … the vibrations of the parts will cross one another, and make a sound … altogether confus'd.[28]

He then turns to the division of the string of the trumpet marine in an effort to demonstrate why certain notes of the brass trumpet are out of tune. He divides the string of the monochord (trumpet marine) into 720 parts—an arbitrary number, but one which allowed him to divide it into smaller parts represented by whole numbers. Assuming the full vibrating length of the string to yield the note c^2, dividing the length in half will yield 360 parts, or the note c^3; dividing it into three equal parts will yield 240 parts, or g^3. The remaining notes, up to the sixteenth harmonic, are shown in a table provided by Robartes (see Figure 10). He then proceeds to show how the seventh, eleventh, thirteenth, and fourteenth notes in the series are out of tune according to just intonation. There are a few flaws in Robartes's calculations, which appear to result from rounding.[29] He surely was aware of these flaws, for he cites the correct values for these notes in the bottom section of his table, using fractions, and also in his representation of the full length of a vibrating string (Figure 11).

Finally, Robartes relates his division of the string of the trumpet marine to the trumpet, stating that the latter's

> notes are produced only by the different force of the breath; it is reasonable to imagine that the strongest blast raises the sound by breaking the Air within the Tube into the shortest vibrations, but that no Musical sound will arise unless they are suited to some aliquot part, and so by reduplication exactly

measure out the whole length of the Instrument…. To which if we add that a Pipe, being shortned [*sic*] according to the Proportions we even now discours'd of in a String, raises the sound in the same degrees, it renders the case of the Trumpet just the same with the Monochord [i.e., trumpet marine].

For a corollary to this Discourse, we may observe that the distances of the Trumpet Notes ascending, [can be] continually decreased in proportion of 1/1, 1/2, 1/3, 1/4, 1/5 *in infinitum*.[30]

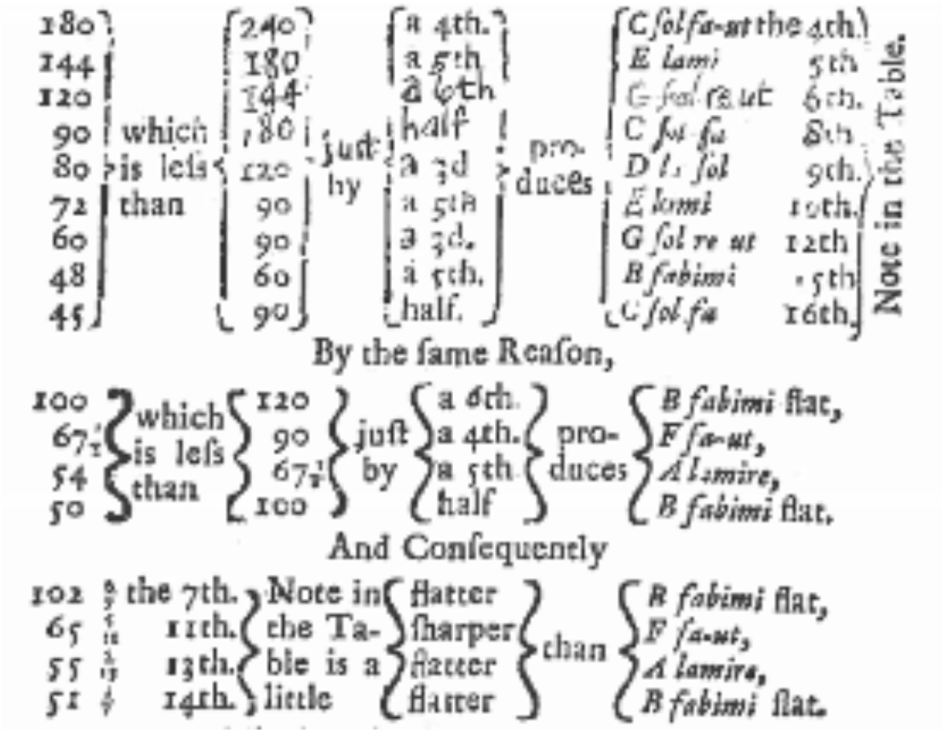


Figure 10: Chart showing why the 7th, 11th, 13th, and 14th harmonics are not in tune. Robartes, "Discourse," unnumbered page after 563.

Figure 11: Robartes, "Discourse," unnumbered page after 563 (illustration rotated 90 degrees counter-clockwise from the original)

I shall return presently to Robartes's reference to the infinite nature of the fractions relating to trumpet notes.

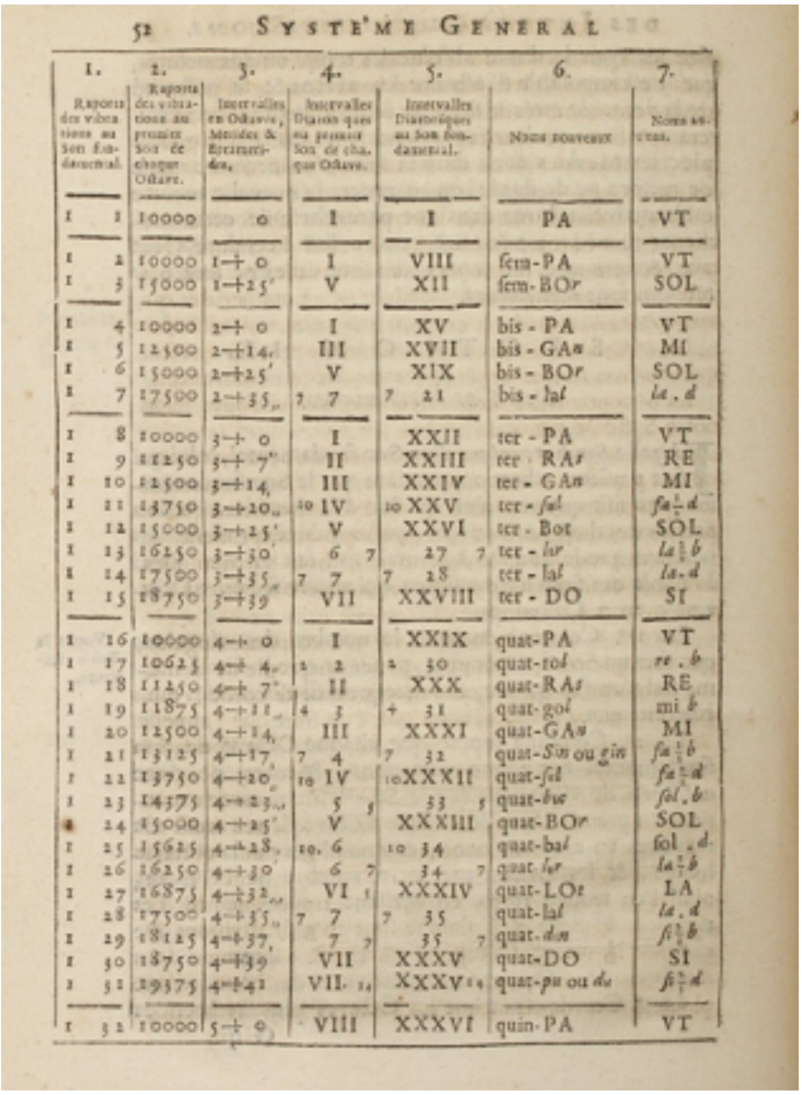
Joseph Sauveur

Building primarily on the work of Mersenne and Noble, Robartes offered the earliest cogent description of the harmonic series, though he did not give it that name and certainly did not realize all of its implications. Just a few years later Joseph Sauveur in France took another step in the development of the theory of the harmonic series. Sauveur established nomenclature that is still current, referring to the science of music as *acoustique,* identifying stationary points on vibrating strings as *noeuds* (nodes), and referring to natural harmonics of a given *son fondamental* as *sons harmoniques.*[31] Some modern writers contend that Sauveur discovered nodes and harmonics independently, though it is difficult to believe he was unaware of the work of the Oxfordians on nodes, as well as that of Robartes, since Sauveur too used the trumpet and the trumpet marine as case studies.[32]

Sauveur's remarks on the trumpet marine, the ordinary trumpet, and other wind instruments are to be found in Section X of his *Principes de musique et d'acoustique* (1701):

> **Section X**
> Application of the General System to the *Trompette marine,* the *Cor de chasse,* and to Large Wind Instruments
>
> This principle of *Sons harmoniques* [i.e., harmonics] having been unknown until now, one should not be surprised if the explanations of the sounds of the *trompette marine,* and of instruments on which the sounds go by leaps, have been imperfect and this discovery will give rise to others for the perfection of [the science of] acoustics….
>
> The *Trompette marine* has a large gut string that passes over a bridge, whose foot is supported by the belly of the *Trompette* [*marine*] and the other [foot] is a little in the air, so that the vibrations of the string are given blows by the foot of the bridge against the belly, which produces the rough sound of the *Trompette marine.*
>
> The thumb, which is pressed down along the string of the *Trompette marine,* acts like a light obstacle and then passes over the divisions of the aliquot and even the aliquant parts,[33] it forms a *Son harmonique*, so that the *Trompette marine* produces only those *Sons harmoniques* up to the 16th [harmonic] in the table in Section IX [see Figure 12], on which are marked the intervals and the names of the notes.
>
> This same table shows also the tones of the ordinary trumpet, the *Cor de chasse,* and the leaps of all wind instruments with [finger]holes. [34]

Sauveur states here that the trumpet marine can play only as high as the sixteenth harmonic and implies that the same is true of the ordinary trumpet and other wind instruments. However, the chart in Section IX of Sauveur's *Principes* (Figure 12) clearly shows pitches ascending to the thirty-second harmonic. Moreover, in a report of a paper he read before the Académie royale des sciences in 1701, we read, "Mr. Sauveur has judged from experience that [the sound of] a string of three feet [in length] can be heard only up to the thirty-second harmonic."[35]



52 SYSTÊME GENERAL

1. Raports des vibrations au Son fondamental.		2. Raports des vibrations au premier Son de chaque Octave.	3. Intervalles en Octaves, Merides & Eptamerides.	4. Intervalles Diatoniques au premier Son de chaque Octave.	5. Intervalles Diatoniques au Son fondamental.	6. Noms nouveaux	7. Noms anciens.
1	1	10000	0	I	I	PA	VT
1	2	10000	1 + 0	I	VIII	sem-PA	VT
1	3	15000	1 + 25'	V	XII	sem-BOr	SOL
1	4	10000	2 + 0	I	XV	bis-PA	VT
1	5	12500	2 + 14,	III	XVII	bis-GAn	MI
1	6	15000	2 + 25'	V	XIX	bis-BOr	SOL
1	7	17500	2 + 35,,	7 7	7 21	bis-lal	la . d
1	8	10000	3 + 0	I	XXII	ter-PA	VT
1	9	11250	3 + 7''	II	XXIII	ter-RAs	RE
1	10	12500	3 + 14,	III	XXIV	ter-GAn	MI
1	11	13750	3 + 20,,	10 IV	10 XXV	ter-fal	fa ½ d
1	12	15000	3 + 25'	V	XXVI	ter-Bor	SOL
1	13	16250	3 + 30'	6 7	27 7	ter-lur	la ½ b
1	14	17500	3 + 35,,	7 7	7 28	ter-lal	la . d
1	15	18750	3 + 39	VII	XXVIII	ter-DO	SI
1	16	10000	4 + 0	I	XXIX	quat-PA	VT
1	17	10625	4 + 4,	2 2	2 30	quat-rol	re . b
1	18	11250	4 + 7'	II	XXX	quat-RAs	RE
1	19	11875	4 + 11,,	4 3	4 31	quat-gol	mi b
1	20	12500	4 + 14,	III	XXXI	quat-GAn	MI
1	21	13125	4 + 17,	7 4	7 32	quat-Sin ou gin	fa ½ b
1	22	13750	4 + 20,	10 IV	10 XXXII	quat-fal	fa ½ d
1	23	14375	4 + 23,,	5 5	33 5	quat-bur	sol . b
1	24	15000	4 + 25'	V	XXXIII	quat-BOr	SOL
1	25	15625	4 + 28,	10 6	10 34	quat-bal	sol . d
1	26	16250	4 + 30'	6 7	34 7	quat-lur	la ½ b
1	27	16875	4 + 32,,	VI 5	XXXIV	quat-LOr	LA
1	28	17500	4 + 35,,	7 7	7 35	quat-lal	la . d
1	29	18125	4 + 37,	7 7	35 7	quat-dan	si ½ b
1	30	18750	4 + 39	VII	XXXV	quat-DO	SI
1	31	19375	4 + 41	VII 14	XXXV 14	quat-pu ou du	si ½ d
1	32	10000	5 + 0	VIII	XXXVI	quin-PA	VT

Figure 12: Joseph Sauveur, Joseph Sauveur, *Principes d'acoustique et de musique, ou systême general des sons* (Paris, 1701), 52.
Translations of the column headings: (1) Relationship of the vibrations to the fundamental tone. (2) Relationship of the vibrations to the first sound in each octave. (3) Intervals in octaves, *merides,** and *eptamerides.*** (4) Diatonic intervals [in relation] to the first note of each octave. (5) Diatonic intervals [in relation] to the fundamental. (6) New names.*** (7) Old names.
*A *meride* is 1/43 of an octave, based on a logarithmic calculation.
**An *eptameride* is 1/7 of a *meride*.
***Sauveur's unique system of names for pitches.

Conclusion

Sauveur placed limits on the harmonic series, though for Robartes it was infinite. There was in fact a resurgence in interest in the mathematical concept of infinity in late-seventeenth-century England. The concept had been known to some extent for centuries: Aristotle commented on it, as did ancient Jain mathematicians in India.[36] But around the time Robartes was working on the natural notes of the trumpet and the trumpet marine, at least a few English mathematicians were involved in the study of this concept, among them John Wallis, who surely was acquainted with Robartes, and also the mathematician, physicist, and astronomer Edmond Halley (of "Halley's Comet" fame). Halley's paper entitled "An Account of the Several Species of Infinite Quantity, and of the Proportions they Bear to One Another" was read before the Royal Society in 1692 and published in that organization's *Transactions* immediately following Robartes's paper on trumpet notes.[37] As an amateur mathematician in his own right, it is not surprising that Robartes took up the concept himself.

Robartes's rather modest objectives were to explain why the natural trumpet and trumpet marine are unable to play certain notes and why some of the notes they *can* play are out of tune. Roger Matthew Grant, however, sees in Robartes's work larger implications for the relationship between music and mathematics.

> The application of an infinite series to pitch by Wallis's colleague Robartes, then, can be understood as a part of the transformation of the concept of number. As continuous and symbolic, numbers could now express infinite geometric or sonic procedures. Furthermore, Robartes's use of the series demonstrated how the concept of fraction [was] transformed with new uses for number…. Robartes's statement of the harmonic series was as much a revolution of the concept of number as it was a contribution to our understanding of the chord of nature.[38]

Titelouze and Mersenne took the first "giant steps" in the discovery of the harmonic series. Four Oxford scholars contributed the discovery of nodes in vibrating strings. Sauveur extended the series to the sixteenth or thirty-second harmonic. Robartes, however, was the first scholar to state that the harmonic series—or at least in his view, the series of notes on the trumpet and the trumpet marine—is infinite, though in practical terms that may be something of an exaggeration. Perhaps the most remarkable aspect of these important early advances in our understanding of the properties of musical sound is that their experiments were decidedly "low-tech"; they employed very simple materials and basic mathematical concepts.

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Notes

[1] In this article I use the term "harmonic" to refer to a member of the harmonic series. I use the term "partial" only in reference to bells, not all of whose overtones (naturally occurring higher frequencies) correspond to the harmonic series. In order to avoid confusion, however, I generally avoid the term "overtone." In common parlance the "overtone series" is essentially identical to the harmonic series, but the "second harmonic," an octave above the fundamental tone, is actually the "first overtone."

[2] "Profectó non semel oseruaui vnius campane sono 3 partes musicae reddi, bassum qui erat primus, atque proprius sonus, diapente, & diapason; diatessaron etiam & ditonum puto me quandoque distinxisse: quod & in organorum tubis, & in alijs instrumentis, imo & in voce obseruari posse credidero." Marin Mersenne, *Quaestiones celeberrimae in Genesim* (Paris, 1623), col. 1699. Transl. Robert W. Ulery.

[3] Concerning the acoustical characteristics of bells, see Arthur Lynds Bigelow, *The Acoustically Balanced Carillon: Graphics and the Design of Carillons and Carillon Bells* (Princeton, NJ: Department of Graphics, School of Engineering, Princeton University, 1961).

[4] "[1] Pourquoi une corde d'instrument sans estre pressee du doigt pour l'accourcir fait divers sons, et les instruments à vent sans ouvrir ni fermer aucun trou? [2] Pouquoy tant les instruments à corde que ceux à vent, sans presser la corde sur la manche, ni ouvrir ou fermer aucun trou, sautant sous son grave à l'octave premierement, puis après à la douziesme, puis à la quinsiesme? [3] Pourquoy en la trompette vers la basse ou son grave, on ne peut entonner *ut, re, mi, fa, sol?*" Paris, Bibliothèque nationale, fonds français, nouv. Acq. 6206, pp. 18–20; transcribed in *Correspondance du P. Marin Mersenne, religieux minime*, ed. Cornelis de Waard (Paris, Presses Universitaires de France, 1955), 565–72. The questions cited in the main text are questions 2–4 from Deschamps's letter. The writer is probably Théodore Deschamps, but the letter is signed only with his family name. These questions were originally posed by Mersenne and repeated in Deschamps's reply. Mersenne's collected correspondence contains only letters that he received; there are apparently no copies of Mersenne's autograph letters.

[5] "Determiner pourquoy vne chorde touché à vuide fait plusieurs sons en mesme temps." Marin Mersenne, *Harmonie universelle contenant la théorie et la pratique de la musique* [hereafter *HM*] (Paris, 1636; facs. rpt., Paris: Centre National de la Recherche Scientifique, 1965), 3:208.

[6] Mersenne refers here to Aristotle's *Problemata*, Problem 8, Section 19, and also Problems 12 and 13.

[7] "Il semble qu'Aristote a cogneu cette experience, lors qu'il a fait la question pourquoy le son graue contient l'aigu…. Mais il faut remarquer qu'il n'a pas sceu que la chorde frappée, & sonnée à vuide fait du moins cinq sons differens en mesme temps, dont le premier est le son naturel de la chorde, qui sert de fondement aux autres….

"Or ces sons suiuent la raison de ces nombres 1, 2, 3, 4, 5, car l'on entend quatre sons differens du naturel, dont le premier est à l'Octave en haut, le second à la Douziesme, le 3 à la Quinziesme, & le 4 às la Dix-septiesme maieure comme l'on void par lesdits nombres qui contiennent les raisons de ces consonances en leurs moindres termes….

"Outre ces quatre sons extraordinaires, i'en entends encore vn cinquiesme plus aigu, que i'oy particulierement vers la fin du son naturel, & d'autresfois vn peu apres le commencement: il fait la Vingtiesme maieure auec le son naturel, auec lequel il est comme troi à vingt." *HM*, 208–09.

[8] The most comprehensive study of the trumpet marine is Cecil Adkins and Alis Dickinson, *A Trumpet by any other Name: A History of the Trumpet Marine*, 2 vols. (Buren: Frits Knuf, 1991).

[9] "[C]es sauts & ces points, qui imitent les sons de la Trompette militaire, ne font autre chose que d'expliquer en grand volume ce que la chorde fait estant touché à vuide, à scauoir l'Octave, la Douziesme, la Quinziesme, la Dix-septiesme, la Dix-neufiesme, &c. les vnes apres les autres, … les quelles elle fait toutes en ensemble en mesme temps." *HM*, 3:221.

[10] Michael Praetorius illustrates a *Trumscheidt* (trumpet marine) with four strings, the three non-melody strings apparently used for a complex system of drones. See *Syntagma musicum*, Bd. 2, *De Organographia* (Wolfenbüttel: Elias Holwein (1619), 57–59, and *Sciagraph*, pl. XXI. I am grateful to Herbert W. Myers for his insights on this matter.

[11] I am grateful to Herbert W. Myers for this observation.

[12] I.e., Jehan Titelouze (ca. 1562–1633), the leading French organist of his day.

[13] "Vnam hîc referam, quam I Titelouzius … alter circa tubam obferuauit, quâ si quis ita cecinerit, vt grauissimum sonum edat, si postea canere velit acutiùs, per diapafon, id est 8 sonis altiùs, quàm antea; si octauam superare velit, per diapente: si diapason diapente supergreditur, id per diatessaron, deinde per ditonum, semiditonum, tonum, & semitonium fiat; vnde concludendum existimat istorum interuallorum consequentiam, & ordinem ex ipsius naturae sinu depromi." Marin Mersenne, *Quaestiones celeberrimae in Genesim* (Paris, 1623), col. 1699. Transl. Robert W. Ulery.

[14] Titelouze appears to have been determined to describe what we know as the harmonic series in terms of intervals of continually decreasing size.

[15] Murray Campbell, Joël Gilbert, and Arnold Myers, *The Science of Brass Instruments* (Heidelberg: Springer International, forthcoming [2021]). I am grateful to the authors of this book for allowing me to quote this passage from it.

[16] Robert Plot, *The natural history of Oxford-shire being an essay toward the natural history of England* (Oxford: The Theater, 1677), 288–99 and 328.

[17] See *John Wallis: Writings on Music*, ed. David Cram and Benjamin Wardhaugh (London: Routledge, 2017), 9–10.
[18] Anthony à Wood, *Athenae oxonienses. An exact history of all the vvriters and bishops who have had their education in the most ancient and famous University of Oxford* (London: for Tho. Bennet, 1691–92), col. 881.
[19] Plot, *The natural history of Oxford-shire*, 296. I have added a few punctuation marks to this quotation for the sake of clarity.
[20] Ibid., 297. I have added a few punctuation marks to this quotation for the sake of clarity.
[21] The performer was probably the elder Jean-Baptiste Prim, one of the leading exponents of the trumpet marine in France in the eighteenth century. See Samuel Pepys, *The Diary of Samuel Pepys*, ed. Robert Latham and William Matthews (London: The Folio Society, 2003), 8:500, cited in Cecil Adkins and Alis Dickinson, *A Trumpet by any other Name: A History of the Trumpet Marine* (Buren: Frits Knuf, 1991), 1:50–51.
[22] [Francis North], *A philosophical essay on musick directed to a friend* (London: printed by John Mertyn for the Royal Society), 18–19.
[23] Since he was a member of Parliament, with some breaks in service, from 1673 until his death in 1718, we know quite a bit about his political career, but relatively little about his musical achievements. Roger North remarked that "none came so neer [to the style of Jean-Baptiste Lully] as the honourable and worthy virtuoso, Mr. Francis Roberts," quoted in *Roger North's The Musicall Grammarian 1728*, ed. Mary Chang and Jamie C. Kassler (Cambridge: Cambridge University Press, 1990), 261–62.
[24] Francis Robartes, "Discourse Concerning the Musical Notes of the Trumpet, and Trumpet Marine, and the Defects of the Same," *Philosophical Transactions of the Royal Society* 16 (1692), 559–63.
[25] Ibid., 559. The lower-case letters *f* and *s* above the staff refer to "flat" and "sharp," respectively.
[26] Ibid.
[27] Ibid., 560.
[28] Ibid., 561.
[29] In the middle layer of the table, using his arbitrary value of 720 for the entire length of a string, Robartes gives 100 as the value for B-flat, but 720 divided by 7 yields a number that is not an integer, which he rounds down to 102. Robartes was obviously aware of this rounding error, for he gave the correct value in the lowest layer of the table, 102 6/7. See Roger Matthew Grant, "*Ad infinitum*: Numbers and Series in Early Modern Music Theory," *Music Theory Spectrum* 35 (2013): 62–76, here 68.
[30] *Philosophical Transactions* 16:563.
[31] Joseph Sauveur, *Principes d'acoustique et de musqiue, ou système general des intervalles des sons….* (Paris, 1701; facs. rpt., Geneva: Minkoff, 1973), 1, 51, and 54, respectively. The papers Sauveur presented to the French Académie royale des sciences were published in the Académie's *Mémoires* for the years 1701–13 (Paris, 1704–16).
[32] Ibid., 58.
[33] As mathematical terms, "Aliquot" and "aliquant" are rather outdated. According to the *Oxford English Dictionary*, "The term 'aliquot parts' refers to whole-number fractions contained in a

larger number a certain number of times without leaving any remainder, forming an exact divisor; "aliquant parts" means "Contained in a larger number but not dividing it exactly." https://www.oed.com/view/Entry/5068?rskey=RQBoeR&result=1#eid and https://www.oed.com/view/Entry/5067?redirectedFrom=aliquant#eid (both accessed 22 June 2020).

34 Sauveur, *Principes d'acoustique*, 58.

"Section X / Application de la Systême general à la Trompette marine, au Cor de chasse, et aux grands instrumens à vent. / Ce principe de Sons harmoniques ayant esté inconnus jusqu'à present, il ne faut pas s'estonner si les les explications des Sons de la Trompette marine, et des Instrumens, dont les Sons vont par saults, ont esté imparfaites; & cette découverte donnera lieu d'autres pour la perfection de l'Acoustique, & même pour trouver des Instrumens d'Acoustique qui répondent à ceux qu'on estime le plus dans l'Optique.

"La *Trompette marine* est composée d'une grosse corde de boyaux, qui porte sur un chevalet, dont un pied est apuyé sur la table de la Trompette, & l'autre est un peu en l'air, ensorte que les vibrations de la corde font donner des coups par la pied du chevalet contre la table qui produisent ce Son aigre de la Trompette marine.

"Le pouce, qui s'aplique le long de la corde de la Trompette marine, tient lieu de l'obstacle leger, & lorsqu'il passe sur les divisions des parties aliquotes et même aliquantes, il se forme un Son harmonique, de sorte que la Trompette marine ne produit que les Sons harmoniques jusq'au le 16e. de la Table de Section IX. dont laquelle sont marquez les Intervalles & les noms de ces Sons.

"Cette même Table marque aussi les Sons de la Trompette ordinaire, du Cor de chasse & les ressaults aux Instrumens à vent qui ont des trous."

35 "M. Sauveur a jugé par experience qu'une corde de 3 piés ne peut guère fair entendre que jusq'à son 32me son harmonique." *Histoire de l'Académie royale des sciences, MDCCI* (Paris: Gabriel Martin, Jèan-Bapt. Coignard & Hippolyte-Louis Guerin, 1743), 121–37, here 136.

36 Regarding Aristotle, see Wolfgang Achtner, "Infinity as a Transformative Concept in Science and Theology," in *Infinity: New Research Frontiers*, ed. W. Hugh Woodfin and Michael Heller (Cambridge: Cambridge University Press, 2014), 19–54, here 21, 23; on Jainists, see George Gheverghese Joseph, *Indian Mathematics: Engaging with the World, from Ancient to Modern Times* (London: World Scientific Publishing, 2016), 101, 115, and 125.

37 E[dmond] Halley, "An Account of the Several Species of Infinite Quantity, and of the Proportions they Bear to One Another," *Transactions of the Royal Society* (1683–1695), vol. 16 (1686–1692), 556–58.

38 Roger Matthew Grant, "*Ad infinitum*: Numbers and Series in Early Modern Music Theory," *Music Theory Spectrum* 35/1 (spring, 2013), 62–76, here 69–70.